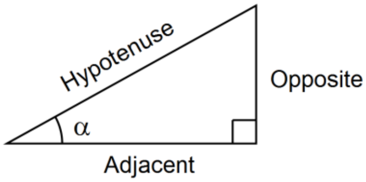
Right Triangle Trigonometry

The six trigonometric functions are the sine (sin), cosine (cos), tangent (tan), cosecant (csc), secant (sec), and cotangent (cot) functions. There are several ways to define these functions of trigonometry. One of the most common mnemonic devices is SOH-CAH-TOA.



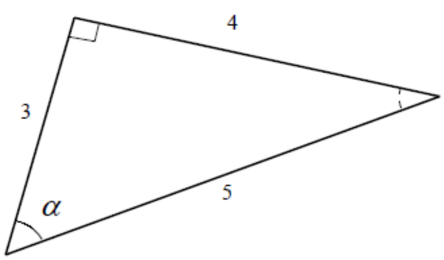
$$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\alpha = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$$

$$\csc\alpha = \frac{\text{hypotenuse}}{\text{opposite}} \qquad \sec\alpha = \frac{\text{hypotenuse}}{\text{adjacent}} \qquad \cot\alpha = \frac{\text{adjacent}}{\text{opposite}}$$

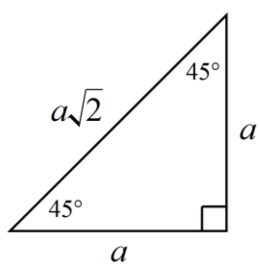
Reciprocal Identities: $\csc\alpha = \frac{1}{\sin\alpha}$ $\qquad \sec\alpha = \frac{1}{\cos\alpha}$ $\qquad \cot\alpha = \frac{1}{\tan\alpha}$

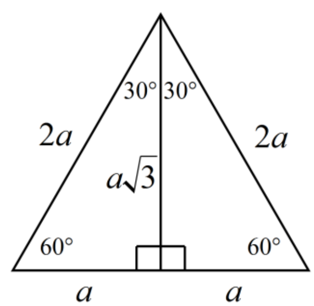
Examples:

Use the following triangle to evaluate $\sin(\alpha), \cos(\alpha), \tan(\alpha), \csc(\alpha), \sec(\alpha),$ **and** $\cot(\alpha)$.



Ratios for Special Right Triangles



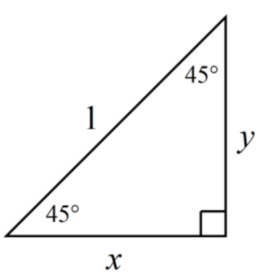


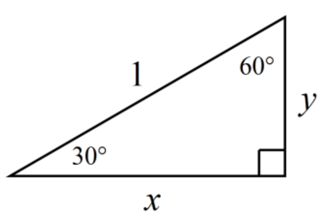
$\text{hypotenuse} = \text{leg} \cdot \sqrt{2}$

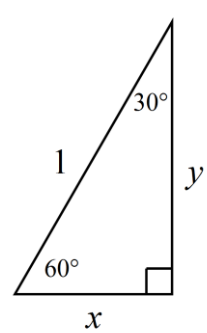
$\text{hypotenuse} = 2 \cdot \text{short leg}$

$\text{long leg} = \text{short leg} \cdot \sqrt{3}$

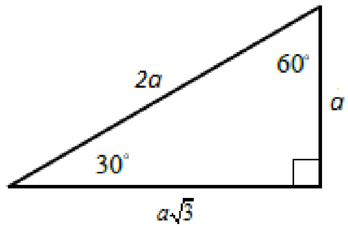
Find the values of x and y in the triangles below:

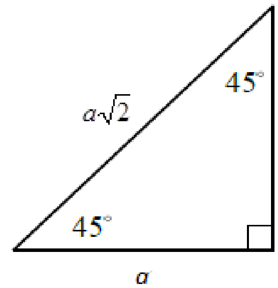






Find the values of the six trigonometric functions for 30° and 45° using the triangles below.





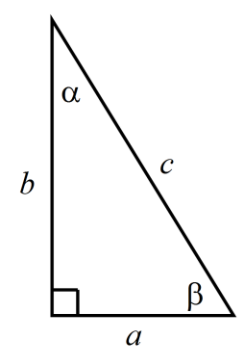
After rationalizing denominators and adding trigonometric functions for 60°, we summarize the trigonometric function values for these special cases in the following table.

Trigonometric Function Values for 30°, 45° and 60°

θ	$\sin(\theta)$	$\cos(\theta)$	$\tan(\theta)$	$\csc(\theta)$	$\sec(\theta)$	$\cot(\theta)$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

Solving Right Triangles

Finding all the missing angle measures and side lengths of a triangle is called "solving a triangle". In a right triangle, we usually name the acute angles α and β (beta) and the lengths of the sides opposite those angles a and b, respectively. The 90° angle is γ (gamma) and the length of the side opposite the right angle (the hypotenuse) is c.

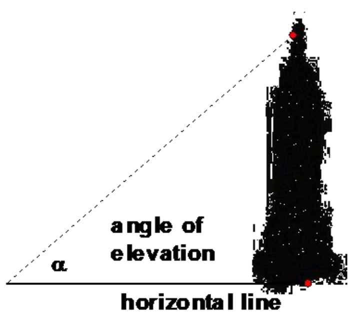


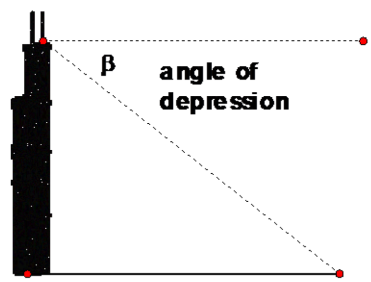
- If you know the lengths of two of the sides, use the Pythagorean Theorem to find the length of the third side.
- If you know the measure of one of the acute angles, use the fact that the angles in a triangle add to 180° to find the measure of the other angle.
- If you know the measure of one angle and the length of one side, use sin, cos, or tan to figure out the lengths of the other sides.

Examples:
Solve the right triangle in which $\alpha = 60°$ and $c = 2$.

Solve the right triangle in which $\beta = 20°$ and $b = 15$.

Using trigonometry, we can find the size of an object without actually measuring the object. Two common terms used in this regard are **angle of elevation** and **angle of depression**.





Examples:
The angle of elevation of the top of a cell phone tower is 38.2° at a distance of 344 feet from the tower. What is the height of the tower?

At one location, the angle of elevation of the top of an antenna is 44.2°. At a point that is 100 feet closer to the antenna, the angle of elevation is 63.1°. What is the height of the antenna?